Name:___________________________ Date:__________________________

Math Club: Logarithmic Functions Worksheet #3

1. Solve each of the following:

$\log(6-x)-2\log x=0$	$\log_3 x^3-\log_3 3x=3$	$\log_2(x-3)+\log_2(x+1)=5$
$\log_7(x+1)+\log_7 x=\log_7 12$	$\log_4(16x-64)-\log_4(3x-34)=3$	$\log(x-7)+\log(x+2)=1$
$\log_3(3x+2)+\log_3 x=\log_3 56$	$\log x=\frac{2}{3}\log 27+2\log 2-\log 3$	$2\log_4 x+\log_4(x-2)-\log_4 2x=1$
$\log_2 16^{2x+1}=8$	$(\log_8 a)(\log_a 3a)(\log_{3a} x^2)=\log_a a^5$	$2\log(3-x)=\log 2+\log(22-2x)$
$\left(\sqrt{x}\right)^{\log x}=100$	$\log_5(x+3)+\log_5(x-1)=1$	$2^{\log x^2}=3\left(2^{1+\log x}\right)+16$

2. Given each systems of equation, find all sets of real numbers (x, y) that satisfies it:

$\log_x 4 + \log_y \sqrt{3} = 5$ $\log_x 8 - \log_y 9 = 13$	$(3x)^{\log 3} = (6y)^{\log 6}$ $6^{\log x} = 3^{\log y}$
$\log_8 4x - \log_8 y = \frac{4}{3}$ $4^{\log_4(4x-y)} = \log_2 16$	$28y^4 = x^2 + 3$ $\log_x y^2 = \log_{y^2} x$
$\log x - \log 3y = 1$ $3^{3x+y} = 27$	Solve the following system of equation: $\log x^3 + \log y^2 = 11$ and $\log x^2 - \log y^3 = 3$ Euclid 2003

3. Find all "x" such that $\log_2(x+2) + 5 = 8 + \log_2 x$

4. Find all "x" such that $2\log x - \log(24 - x) = \log 2$

5. Solve for "x" $\log_2(2x+4)-\log_2(x-1)=3$

6. Determine the exact value(s) of all "x" such that: $\log_4 x+\log_2\sqrt{x-2}=1+\log_{16}(x-1)^2$

7. Solve for "x": $\log_a b+\log_{a^2} b=\log_{a^3} b^x$

8. Determine all value(s) of "x" that satisfy the equation (find all the extranneous roots) $\log_{5x+9}(x^2+6x+9)+\log_{x+3}(5x^2+24x+27)=4$

9. If $\log_2 x$, $(1+\log_4 x)$, $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible value(s) of "x". Euclid 2009

10. Given that $\log\sin x+\log\cos x=-1$ and that $\log(\sin x+\cos x)=0.5(\log n-1)$, find the value of "n". 2003 AIME

11. Determine the number of ordered pairs (a,b) of integers such that $\log_a b + 6\log_b a = 5$, with $2 \le a \le 2005$, and $2 \le b \le 2005$. 2005 AIME

12. Solve the following system of equations: Note There are two solutions: AMC 2002

$\log_{225} x + \log_{64} y = 4$ & $\log_x 225 - \log_y 64 = 1$

13. Determine all real solutions to the system of equations and prove that there are no more solutions: Euclid 2008

$$\begin{aligned} x + \log(x) &= y - 1 \\ y + \log(y-1) &= z - 1 \\ z + \log(z-2) &= x + 2 \end{aligned}$$

14. Let: $\begin{aligned} S_1 &= \{(x,y) \mid \log(1 + x^2 + y^2) \le 1 + \log(x+y)\} \\ S_2 &= \{(x,y) \mid \log(2 + x^2 + y^2) \le 2 + \log(x+y)\} \end{aligned}$. What is the ratio of the area of S_2 to the area of S_1? 2006 AMC 12A

15. Let "S" be the set of ordered triples (x,y,z) of real numbers for which:

$\log(x+y) = z$ and $\log(x^2 + y^2) = z + 1$.

There are real numbers 'a" and "b" such that for all ordered triples (x,y,z) in "S" have $x^3 + y^3 = a \times 10^{3z} + b \times 10^{2z}$. What is the value of $a + b$? AMC 2005 12B

Euclid 1999

(b) Determine the coordinates of the points of intersection of the graphs of $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$.

23. Let S be the set of ordered triples (x, y, z) of real numbers for which

$$\log_{10}(x+y) = z \quad \text{and} \quad \log_{10}(x^2+y^2) = z+1.$$

There are real numbers a and b such that for all ordered triples (x, y, z) in S we have $x^3 + y^3 = a \cdot 10^{3z} + b \cdot 10^{2z}$. What is the value of $a + b$?

(A) $\frac{15}{2}$ **(B)** $\frac{29}{2}$ **(C)** 15 **(D)** $\frac{39}{2}$ **(E)** 24

9. (a) If $\log_2 x$, $(1 + \log_4 x)$, and $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible values of x.

6. The solutions to the system of equations

$$\log_{225} x + \log_{64} y = 4$$
$$\log_x 225 - \log_y 64 = 1$$

2002 AIME

are (x_1, y_1) and (x_2, y_2). Find $\log_{30}(x_1 y_1 x_2 y_2)$.

4. Given that $\log_{10}\sin x + \log_{10}\cos x = -1$ and that $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, find n.

5. Determine the number of ordered pairs (a, b) of integers such that $\log_a b + 6\log_b a = 5$, $2 \le a \le 2005$, and $2 \le b \le 2005$.

3. A possible angle, x, in radians which satisfies $\log_2(\cos(x)) = -\frac{1}{2}$ is:

   a) $\frac{\pi}{6}$ b) $\frac{2\pi}{3}$ c) $\frac{3\pi}{4}$ d) $\frac{4\pi}{3}$ $*$e) $\frac{7\pi}{4}$

(b) Solve the system of equations:

$$\log_{10}(x^3) + \log_{10}(y^2) = 11$$
$$\log_{10}(x^2) - \log_{10}(y^3) = 3$$

solutions.

(b) Determine all real solutions to the system of equations

$$\begin{aligned} x &+ \log_{10} x &= y - 1 \\ y &+ \log_{10}(y-1) &= z - 1 \\ z &+ \log_{10}(z-2) &= x + 2 \end{aligned}$$

and prove that there are no more solutions.

21. Let

$$S_1 = \{(x, y) \mid \log_{10}(1 + x^2 + y^2) \le 1 + \log_{10}(x + y)\}$$

and

$$S_2 = \{(x, y) \mid \log_{10}(2 + x^2 + y^2) \le 2 + \log_{10}(x + y)\}.$$

What is the ratio of the area of S_2 to the area of S_1?

(A) 98 **(B)** 99 **(C)** 100 **(D)** 101 **(E)** 102

Euclid 2006

8. (a) If $\log_2 x - 2\log_2 y = 2$, determine y as a function of x, and sketch a graph of this function on the axes in the answer booklet.